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Technical Note

Boiling curves – bifurcation and catastrophe

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Abstract

Multi-regime feature of boiling system has ever eluded researchers despite decades of efforts. Basing on catastrophe theory, the topological boiling space was constructed from fundamental nucleating equation. All possible boiling curves are actually the projections of boiling space. It is demonstrated that transition of modes is virtually corresponded to the bifurcations. Catastrophe behaviors and hysteresis in boiling system have then been discussed by basic ideas in catastrophe theory. The effects of thermal physical properties and other parameters, such as liquid velocity, sub-cooled degree and gravity on boiling curves were revealed. The present model is in good agreement with available experimental facts. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The understanding of the different boiling regimes is very important for industrial production. Nukiyama [1] first experimentally gave the consideration of the heat flux versus wall superheat boiling curve and the boiling transition. Over the past decades, many investigations have been conducted to examine the heat flux versus wall superheat boiling curve. However, the essence of the boiling curve is not yet well understood, especially in transition regime from nucleate boiling to film boiling. As is well known, catastrophe and hysteresis phenomena often appear during the transition from nucleate boiling to film boiling [2]. As shown in Fig. 1, obviously, within the scope of *ABCD*, for the existence of multi-fold solutions, system may jump directly from a regime to another one under a certain degree of disturbance.

In fact, there is as yet no consensus as to catastrophe and hysteresis in boiling system. Mechanistic prediction of catastrophe and hysteresis has eluded researchers despite decades of efforts. Recently, catastrophe and hysteresis sometimes take place and sometimes not in previous research, which makes the problem more elusive [2].

In our viewpoints, nonlinear effects are the most essential factors that give rise to catastrophe and hysteresis phenomena in boiling system. Starting from nonlinear interaction effects and catastrophe theory, the present investigation analyzes the transition of boiling modes. In fact, the present investigation is another renewed effort taken to investigate the nonlinear effects from different perspectives from previous researches [3,4]. First, the 'mathematical topological boiling space' constructed by all possible boiling curves which determines the boiling mode transition, is derived basing on fundamental boiling nucleating equation. Catastrophe behaviors and hysteresis in boiling system have then been discussed by basic ideas in catastrophe theory. The effects of thermal physical properties and other parameters, such as liquid velocity, sub-cooled degree and gravity on transition boiling were revealed. The present model is in good agreement with available experimental data and observations.

2. Boiling space basing on catastrophe theory

The catastrophe theory that was first proposed by Thom is one of the most appreciated theories in

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Nomenclature

g	gravitational acceleration (m s^{-2})
h	constant
h_{fg}	latent heat of evaporation (J kg^{-1})
Ja	degree of subcooling
k	constant
n_u	control variable defined in Eq. (3)
N_0	constant
q	heat flux (J m^{-2})
T	temperature (°C)
u	velocity (m s^{-1})
U	potential function
x	state variable (m)

Greek symbols

T	liquid superheat (°C)
σ	surface tension (N m^{-1})
ρ	density (kg m^{-3})
θ	contact angle (°)
μ	control variable
v	control variable

Subscripts

ℓ	liquid
s	saturated state
v	vapor
0	reference state

international academy field [5]. Recently it has found wide applications in many fields. Detailed introductions are neglected here and interested readers may find more information in [5]. Catastrophe theory is actually a potential dynamic theory that describes nonlinear processes. Here basic ideas of catastrophe theory are used to analyze the feature of boiling curves, especially about catastrophe and hysteresis in boiling system.

Cusp-type is the most widely used catastrophe type in engineering, whose potential function in three-dimensional space is

$$U = x^4 + \mu x^2 + \upsilon x, \tag{1}$$

where μ and v are controlling parameters, x is variable.

All critical points for U form the equilibrium face M

$$\nabla U = 4x^3 + 2\mu x + \upsilon = 0. \tag{2}$$

The points on M are singularity points.

As far as boiling process is concerned, the transition for boiling regimes is mainly controlled by superheat, which is here selected as state variable. At low superheat nucleate boiling prevails, while film boiling will dominate at high superheat.

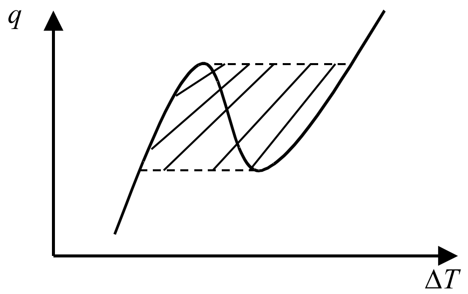

Fig. 1. Catastrophe and hysteresis in boiling system.

Topological method is applied here to generate potential function U for boiling process. Basic equation for nucleating process is [6]

$$\Delta T = \frac{T_s}{\rho_v h_{fg}} \left[\frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 k T_\ell / h)} \right]^{1/2}, \tag{3}$$

where $f(\theta) = (1/4)(2 + 3\cos\theta - \cos^3\theta)$. Eq. (3) can be changed as

$$F(\Delta T) = \Delta T^2 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 k T_\ell / h)}. \tag{4}$$

Assuming there is a topological differential same embryo

$$\Phi : \sqrt{3}(\Delta T - \Delta T_0) \rightarrow \Delta T. \tag{5}$$

Then

$$F(\Delta T) \sim 3(\Delta T - \Delta T_0)^2 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 k T_\ell / h)}. \tag{6}$$

So

$$\frac{\partial^2 U}{\partial \Delta T^2} \sim 3(\Delta T - \Delta T_0)^2 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 k T_\ell / h)}. \tag{7}$$

Eqs. (4)–(7) mean that bifurcation or catastrophe characteristics will not change under topological transformation.

Integrating Eq. (7) yields

$$\frac{\partial U}{\partial \Delta T} \sim (\Delta T - \Delta T_0)^3 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 k T_\ell / h)} \times a(\Delta T - \Delta T_0) + F(q). \tag{8}$$

Since the heat flux must be an indispensable parameter for the boiling process, it is conceptually logical that the integration constant is here assumed to be the function of heat flux.

$F(q)$ can be cast into Taylor series as

$$F(q) = c_0 + c_1 q + \cdots \tag{9}$$

For simplicity, the higher-order terms are neglected, which will not influence the topological characteristics of boiling curves, and let $c_0 = -q_0$, $c_1 = 1$, Eq. (8) is changed as

$$\frac{\partial U}{\partial \Delta T} \sim (\Delta T - \Delta T_0)^3 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 kT_\ell/h)} \times (\Delta T - \Delta T_0) + q - q_0. \tag{10}$$

Integrating Eq. (10) again yields

$$U = \frac{1}{4}(\Delta T - \Delta T_0)^4 - \frac{1}{2}\frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 kT_\ell/h)} \times (\Delta T - \Delta T_0)^2 + (q - q_0)(\Delta T - \Delta T_0), \tag{11}$$

where integration constant is neglected, being that it will not affect the catastrophe features. It is evident that transitions of boiling modes belong to cusp-typed catastrophe.

Eqs. (10) and (11) can produce the topological boiling space and boiling curves. Let Eq. (10) equal zero and get topological equilibrium boiling space

$$(\Delta T - \Delta T_0)^3 - \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 kT_\ell/h)}(\Delta T - \Delta T_0) + q - q_0 = 0. \tag{12}$$

Assuming

$$n_u = \frac{T_s^2}{\rho_v^2 h_{fg}^2} \frac{16\pi\sigma f(\theta)}{3kT_\ell \ln(N_0 kT_\ell/h)} \tag{13}$$

is a variable reflecting the thermal physical properties, Eq. (12) is schematically shown in Fig. 2(a). For same liquid boiling under same conditions, the boiling space can be simplified to single boiling curve as commonly expected, as shown in Fig. 2(a). Within the scope of threefold solutions, as shown in Fig. 2(b), system may jump directly from a regime to another one under a certain degree of disturbance.

3. Results and discussions

3.1. Transitions of boiling regimes

There is yet no consensus as to the mechanism controlling transition from nucleate boiling (NB) to film boiling (FB) [7]. Currently, theoretical analyses of the transition from NB to FB have generally assumed one of two different mechanisms: thermodynamic or hydrodynamic. Here we construct the nonlinear dynamic boiling space. This kind of nonlinear considerations may incorporate both the thermodynamic mechanisms and the hydrodynamic mechanisms, which are in agreement with a series of studies conducted in [8–11].

On topological boiling space as shown in Fig. 2(a), with the increase of superheat, the system may change from NB to FB, passing by folding section (denoting transition boiling). On other hand, with the decrease of superheat, the system may also change from FB to NB, also passing by folding section. The system would transmit from one state to another state only at disturbance with a certain magnitude when superheat reaches a certain value. Obviously, disturbance plays a dominant role in jump from one state to another state in boiling system. The catastrophe phenomenon will not take place until the disturbance can overcome impedance of potential at certain point.

Obviously, the occurrence of boiling modes transitions corresponds to the bifurcation phenomena in nonlinear dynamic systems. When superheat is relatively low, the boiling system prevails at NB regime where there exists two-phase structure: vapor phase and liquid phase. The boiling system evolves thereafter with the increase in superheat until FB (only one vapor phase structure near the wall surface) occurs, suddenly. This is a kind of bifurcation, which also means the

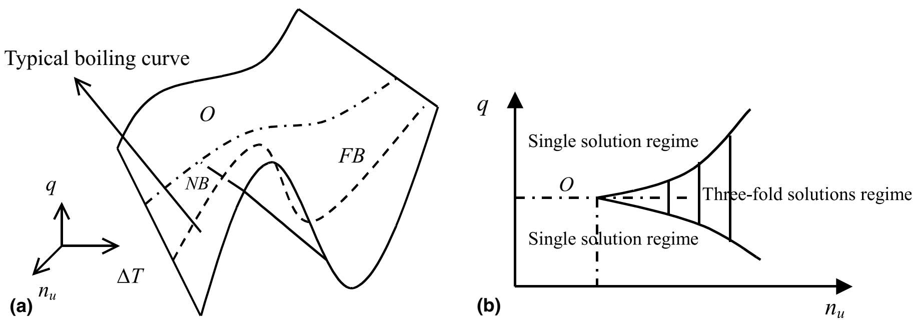

Fig. 2. Topological boiling space: (a) topological boiling space composed by boiling curves; (b) the projecting curve in q–n_u plane.

occurrence of new dissipative structure. The essence of transition in boiling modes is actually mathematically described here.

3.2. Hysteresis during boiling processes

As shown in Fig. 3, obviously, hysteresis phenomena that usually were observed in transition boiling modes are clear here now. When disturbance is small, the superheat that is needed for the transition from NB to FB will greatly increase. On the other hand, the superheat required for the transition from FB to NB will significantly reduce, which means the existence of evident hysteresis.

Catastrophe theory plays a more important role in explaining the evolution and hysteresis between NB to stable FB. The essence of transition from NB to stable FB is the destabilization of liquid–vapor interface. Superheat and disturbance play the most important role in the transition between NB and stable FB. It is not necessary to debate on the existence of catastrophe and hysteresis in the transition process between NB and FB, for they depend on experimental conditions such as heated surface, heating methods and liquid properties. The significance of catastrophe and hysteresis is also dependent on experimental conditions. In other words, the catastrophe and hysteresis in boiling system can be mainly controlled by supervising superheat and disturbance effectively placed.

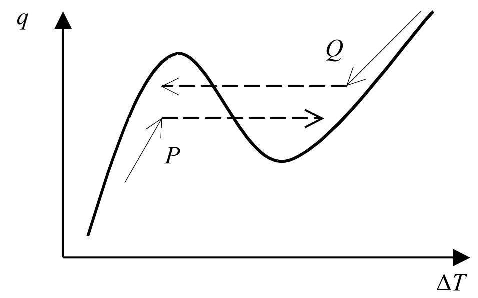

Fig. 3. Catastrophe and hysteresis during boiling process.

In terms of Eq. (11), it is clear that potential U depends on many factors. The effect of contact angle is one important aspect. Disturbance required for inducing jump will be different for different contact angles. Restate, the effect of advancing contact angle and receding contact angle on hysteresis phenomena in transition boiling, which is commonly appreciated in previous research, is clear here. However, it is useful to note that superheat plays a more important role than the difference between the advancing contact angle and the receding contact angle. This may be the reason why significant hysteresis phenomenon exists when the difference between the advancing contact angle and the receding contact angle is neglected [10].

As introduced above, nonlinear effects are the most essential factors that give rise to catastrophe and hysteresis phenomena in boiling system. Here we analyze the catastrophe and hysteresis from the view of mathematical nonlinear potential dynamics.

3.3. The avoidance of unsteady transition boiling regime

It is demonstrated in the experimental results in [12,13] that liquid velocity, sub-cooled degree and gravity have great influence on boiling curves, as shown in Fig. 4(b). Transition phenomena tend to disappear at extremely high liquid velocity, sub-cooled degree and gravity. Here we explain it theoretically. The liquid alternately contacting the heating surface significantly characterizes the nucleating process in transition boiling. Restate, behaviors of liquid–vapor interface greatly affect the characteristic of nucleation in transition boiling. The greater the liquid velocity, the stronger the destabilization of liquid–vapor interface, which results in easier contacting of liquid on heating surface, which will promise the decrease of the constant n_u and in turn the decrease of the transition regime with threefold solutions, as shown in Fig. 4(a).

Gravity has the same effect on transition as velocity. The greater the gravity, the stronger the unstable wave induced, which implies that it produces same effect as increasing velocity as shown in Fig. 4(b). Of course, for

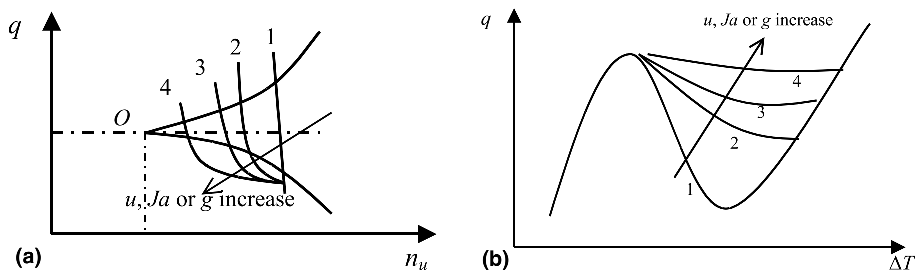

Fig. 4. Illustration of effects of u, Ja and g on transition boiling.

currently there are no experimental results related, this conclusion may be viewed as logical deduction.

As for the effect of sub-cooled degree, it can be explained in same principle. The greater the sub-cooled degree, the less energy the vapor possesses which will give the results of easier destabilization of liquid–vapor interface. Thus same effects as increasing liquid velocity will be produced, as shown in Fig. 4(b).

Another more evident fact is that, for boiling of non-wetting liquid, which means constant n_u is very small, the system may directly change from NB to FB with no occurrence of transition boiling regime [6].

4. Conclusions

Catastrophe theory has been used to analyze the features of boiling curves, especially about catastrophe and hysteresis of boiling modes. Transitions of boiling modes virtually correspond to mathematical bifurcation in topological boiling space. Catastrophe and hysteresis in boiling system have been discussed using basic ideas in catastrophe theory. The effects of thermal physical properties and other parameters, such as liquid velocity, sub-cooled degree and gravity on boiling curves were revealed.

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